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Characteristic Equations for Superheated Steam

Mechanical Engineering

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CHARACTERISTIC EQUATIONS FOR SUPERHEATED STEAM

BY

CLEMENT BERNARD WISSING

THESIS

FOR THE

DEGREE OF BACHELOR OF SCIENCE

IN

MECHANICAL ENGINEERING

COLLEGE OF ENGINEERING

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May 29 1911

THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Clement Bernard Wessing

ENTITLED Characteristic Equations for Superheated Steam

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Bachelor of Science in Mechanical Engineering

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APPROVED: G. G. Goodenough

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197773

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TABLE OF CONTENTS.

Part I.

Summary of Previous Equations.

	Page
Regnault's determinations on saturated steam.	1.
Zeuner's calculations.	1.
Battelli's Experiments on saturated and superheated steam.	1.
Laboratory of Technical Physics at Munich.	2.
Prof. Dr. C.V. Linde's Experimental Apparatus.	2.
Duration of Experiments.	2.
Clapeyron-Clausius Equation for Specific Volume of Steam.	2.
Determination of Specific Volume.	2.
Records of Experiments and difficulties encountered.	3.
Hirn's experiments.	3.
Description of Linde's apparatus and method.	3.
Equation of Knoblauch, Linde and Klebe.	5.
Comparison of results obtained by various experimenters.	5.
Knoblauch, Linde and Klebe's table of specific volumes.	6.

Part II.

Discussion of the New Equation.

Equation proposed by Prof. G.A. Goodenough with constants.	7.
Values of Constants in French and English Units.	8.
Method of obtaining constant volume chart, diagram 1.	8.
Method of obtaining isothermal chart, diagram 2,	8.
Method of investigation of equation.	9.
Effect on isothermals of varying the constants.	9.
Constants finally selected.	10.

Page

Table containing comparative values of slope $(\frac{dp}{dt})_v$. 11.

Specific Volumes with saturation pressures and temperatures. 11.

Derivation of slope equation. 11.

Discussion of new equation. 11.

Part III.

Sample Calculations.

Complete calculations for conversion of constants. 13.

Product of pressure and volume. 16.

Calculations for points on lines by new equation. 17.

Calculations for slope $(\frac{dp}{dt})_v$. 19.

CHARACTERISTIC EQUATIONS FOR SUPERHEATED STEAM.

Part I.

Summary of Previous Equations.

The work involved in the calculations along the lines of the investigations in this paper is based upon the experimental determinations of Knoblauch,, Linde and Klebe.

It will be the purpose, therefore, in Part I of this paper to give a brief summary of the report of these men, in order to allow a clearer discussion of the matter in hand. The purpose of their experiments was to determine experimentally the properties of superheated steam.

The first authoritative determinations of the pressures and temperatures of saturated steam were published by Regnault. Mathematical calculations for the specific volume of saturated steam, based on Regnault's determinations, were made to supplement his statements by Zeuner, who for this purpose made use of the Clapeyron-Clausius equation. About forty-five years after Regnault, Battelli made experiments on the pressure, temperature and specific volume of saturated and superheated steam and extended the limits of pressures and temperatures to the critical point. Battelli's determinations varied considerably from those calculated by Zeuner..

To investigate this condition, a set of experiments

was carried on in the Laboratory of Technical Physics of the Royal Technical High School at Munich. This laboratory was opened in 1902, and Prof. Dr. C.V. Linde placed at its disposal an apparatus built and designed by him for the Linde Ice Machine Company's experimental station, for determining the density of steam. A Gerykschen air pump and a steam boiler supplied with gas burners were the only things necessary to complete the apparatus.

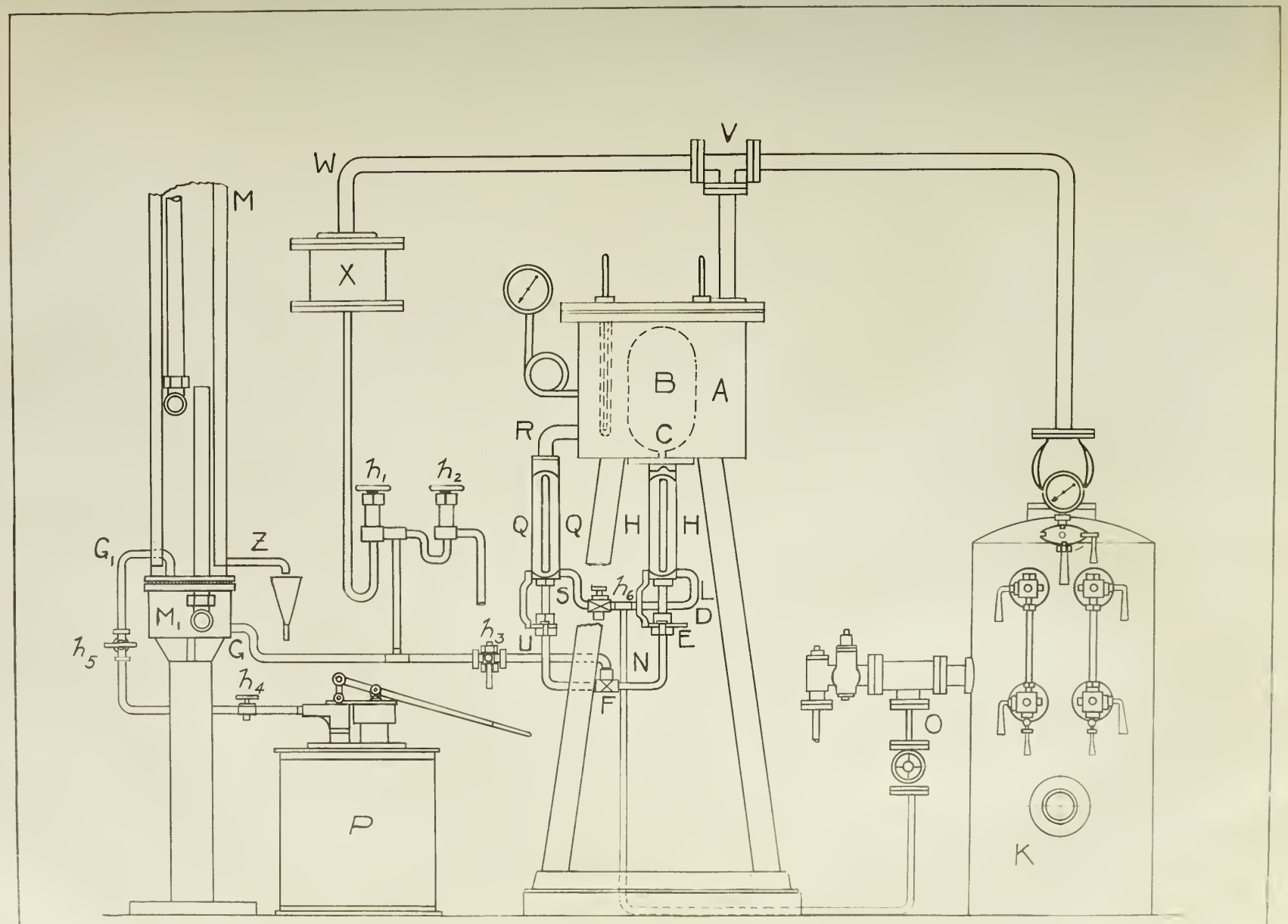
The experiments carried on in the Technical Laboratory extended over three years, due to extensive preliminary work, which was made necessary by the extreme accuracy required in the final results.

The condition of saturated and superheated steam was determined by making use of the pressure (p), volume (v), and temperature (t), or in absolute units (T). For calculating specific volumes the Clapeyron-Clausius equation was used. The statement of this equation is:

$$\frac{r}{u} = AT \frac{dp}{dt}$$

in which "r" is the latent heat of vaporization, "A" the reciprocal of the mechanical equivalent of heat and "p" the saturation pressure. "U" is the difference between specific volume of dry saturated steam and of water at the same temperature.

Determinations of the specific volume of either saturated or superheated steam may be made by taking the observations of corresponding values of pressure, volume and tempera-



PROF. LINDE'S APPARATUS

PLATE 1.

ture, just as condensation in a certain volume of water appears or as condensation completely vanishes. Battelli made experiments using the general principles involved in this method. Herwig, Wüllner and Grotrian, as well as Ramsay and Young, made similar determinations, but the method of procedure was varied somewhat in all cases. The difficulties experienced and the sources of error were in all cases due to condensation in the neighborhood of saturation, absorption of gases by the enclosing vessels and the large ratio between the surface and the enclosed volume.

These factors tended to cause errors which were unavoidable with the apparatus used. Hirn used a larger volume and a brass containing vessel, but this introduced the error due to the absorption of gases by metals. The values of Hirn were smaller than those of Knoblauch, Linde and Klebe, who were enabled to use a glass containing vessel by means of Dr. Linde's design. This design relieved the glass vessel of excessive internal pressure, as is shown in the following description of the apparatus.

See Plate I.

In an iron vessel A is located a glass balloon B. The glass tube CD which is fused onto it, ends at D in a stuffing box and can be connected to the mercury manometer M through the iron tube EFG. The steam from the boiler K, fills the vessel A, and the balloon tube for a distance of 250 mm. The condensation runs back into the boiler through the conduit

LNO, as the boiler and vessel remain under the same pressure. Within the tube QQ is placed a glass tube RS similar to CD and of the same size, which ends at S in a stuffing box, and through the connecting tube UF allows a connection to be made between the heated space A and the manometer M.

A measured amount of water is placed in the balloon and slowly evaporated by the heat in the steam from the boiler. The mercury in the manometer must fill the tube GF, as well as FE at the same time, and must be high enough in SR and DC to allow the surface to be observed through the windows at Q and H. As the heating steam surrounds the balloon tube as far as the mercury, the steam inclosed in the balloon is certain to be of the same temperature as the heating steam. As long as liquid water is present in the balloon, the mercury stands at the same height at Q and H. By comparing Q or H with the mercury level in the manometer, the saturation pressure corresponding to the temperature in A is found. The air pressure must also be considered.

By a further rise in temperature the water in the balloon is entirely evaporated, while the meniscus at H is kept fixed, to insure a constant volume in the balloon.. It is further necessary to take the precaution that the water in the balloon is just sufficient to fill it with saturated steam at the corresponding temperature. When the steam in the balloon superheats the level at Q will fall below that at H, and the comparison with the manometer gives the pressures corre-

sponding to the respective temperatures for saturated and superheated steam.

When now, on increasing the temperature, the meniscii Q and H just begin to change in level, the superheating begins, and also when they exactly resume their levels superheating ceases. The relation then of the enclosed volume in the balloon to the weight of water gives the specific volume.

Now with temperatures as abscissae and pressures as ordinates two point paths were obtained; thus (pt) curves for saturated and superheated steam at constant volume. By extending these lines to the saturation curve, the corresponding saturation pressures and temperatures were found for the existing specific volumes. These constant volume lines were inclined to the temperature line to a greater extent at the lower specific volumes than at the higher. It was further found that all of these constant volume lines intersected at the temperature- 273.

The recorded results of Knoblauch, Linde and Klebe were calculated by means of the equation:

$$pv = BT - p(1+ap)\left[C\left(\frac{373}{T}\right)^3 - D\right]$$

where (p is in $kg/sq\ m$ (v) in cbm/kg and the four constants

B=47.10 a=.000002 C=.031 D=.0052

The values of the specific volume in both the saturated and superheated regions as found by Knoblauch, Linde and Klebe vary 3% from those of Battelli, and by 2% from those of Zeuner who took A as $\frac{1}{424}$, instead of $\frac{1}{427}$ according to later measurements. This makes the deviation from .8 to 1.2 between

Knoblauch, Linde and Klebe and Zeuner.

The values of V vary from those of Regnault by only .4% to .7% inasmuch as the $(\frac{dp}{dt})$ obtained by Knoblauch, Linde and Klebe are smaller than those of Regnault. The values of the specific volume of saturated steam as found by Knoblauch, Linde and Klebe also approach nearer to Regnault-Zeuner than to Battelli. The pressure and temperature determinations of these latter experiments are not assumed by the experimenters to be the best obtainable, on account of a small uncertainty in the thermometers used which, however, amounted to only .1°C. This uncertainty is noticeable to only a very small extent in the superheated region amounting to about .05% error. In the saturated region it is somewhat larger.

The following is a table of the specific volumes as found by Knoblauch, Linde and Klebe and correspond to the constant volume lines as shown on Diagram 1.

Table of specific volumes.

Exp.	V.	Exp.	V.	Exp.	V.
1	1.598	12	.7405	23	.3582
2	1.550	13	.6400	24	.3540
3	1.288	14	.6340	25	.3258
4	1.189	15	.6143	26	.3100
5	1.122	16	.5204	27	.2809
6	1.039	17	.4955	28	.2652
7	1.023	18	.4710	29	.2428
8	.9150	19	.4666	30	.2188
9	.9085	20	.4566	31	.1910
10	.8328	21	.3950	32	.1817
11	.8250	22	.3897		

Part II.

Discussion of the New Equation.

Due to the accuracy with which the experiments of Knoblauch, Linde and Klebe were carried on and the precautions which were taken to insure the very best conditions in the test, we may assume that their determinations for the pressure, volume and temperature relations are the most authoritative and reliable of any existing at the present time. The characteristic equation derived by these experimenters as based on the results of the experiments conducted in the Laboratory of Technical Physics, at Munich, is one of the best before the Technical world at the present time. This equation, as it was before expressed, has the following form:

$$pv = BT - p(1+ap)\left[C\left(\frac{373}{T}\right)3 - D\right]$$

It will be seen that this equation is somewhat complicated in form, and it is the purpose of this investigation to test an equation proposed by Prof. G.A. Goodenough, which is simpler in form and more easily adapted to ordinary calculations. The new equation proposed should be derived from the plotted values of the above mentioned experiments and should, furthermore, involve as little as possible, any change in the slope $\frac{dp}{dt}$.

The new equation:

$$pv = BT - p(1+ap) - \frac{m}{T}n - Cp,$$

although it contains five constants, is much simpler in form, and

therefore, easier to manipulate. The values of the constants in the French Units, as first proposed were:

Constant	French System	English System
B	47.16	85.957
C	.006	.0961
A	.00000085	.00000415
log m	11.19839	13.679376
n	5	5

The calculations required in this investigation may be taken up under four general heads, and this will be done in detail under the Sample calculations in Part III. As the calculations under each head are repeated a number of times, it will be necessary to include only one complete set of figures, to show the method in each case. Only the method of pursuing the work will be given in Part II.

In beginning this work it was first necessary to reproduce, on a large scale, the two curves of Knoblauch, Linde and Klebe. Diagram 1 is a reproduction of the chart showing the constant volume lines on a temperature-pressure basis. This diagram was constructed by plotting directly, the values given in the report of the original experiments.

Diagram 2 is a reproduction of the isothermal chart, with pressures as abscissae and the products of pressures and volumes as ordinates. The points for this diagram were obtained by calculating the product according to the method described in Part III, with values for pressures and volumes taken from the tables of Knoblauch, Linde and Klebe. Diagram 2 was used to compare the results calculated by means of the new characteris-

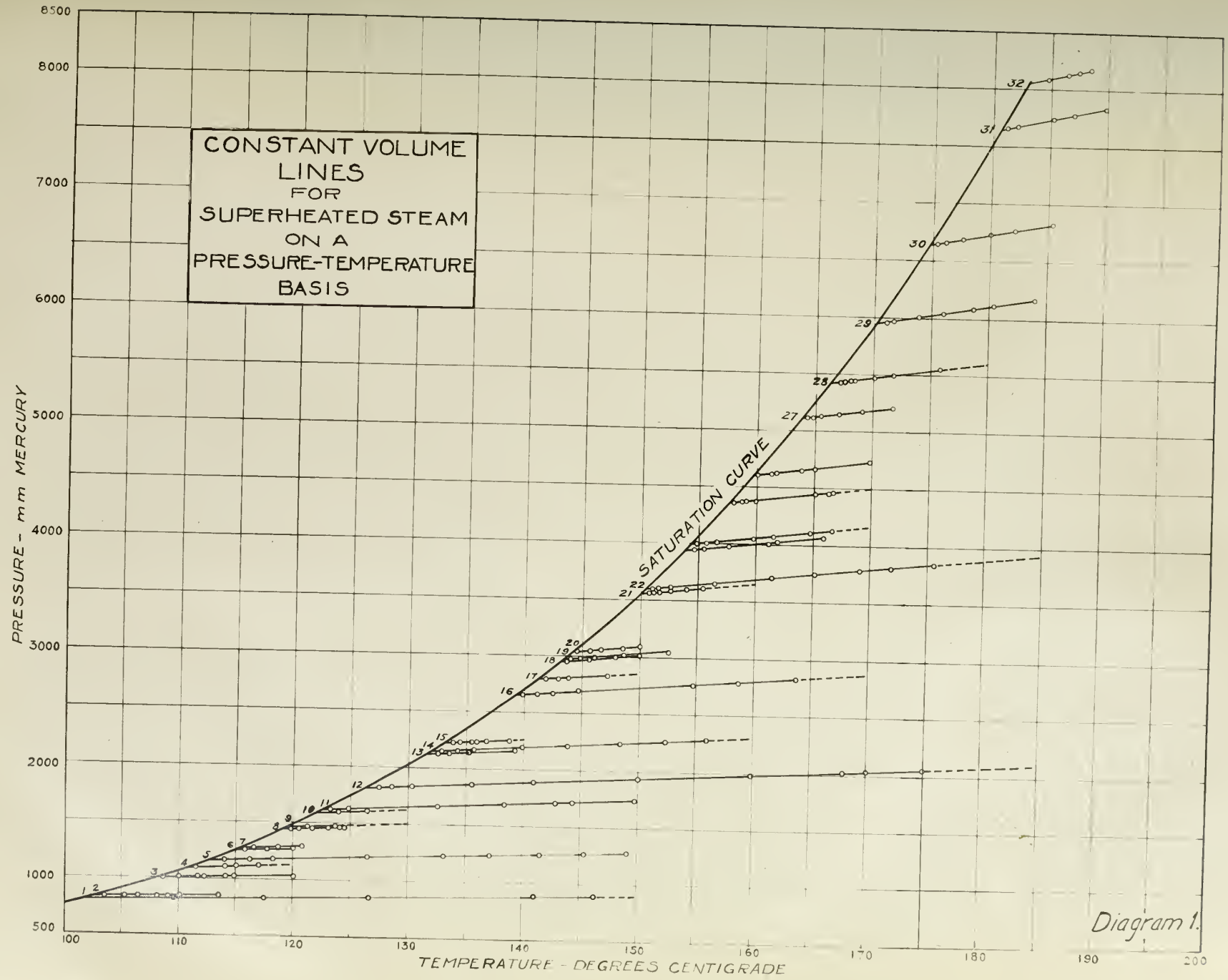

CONSTANT VOLUME LINES FOR SUPERHEATED STEAM ON A PRESSURE-TEMPERATURE BASIS
SATURATION CURVE
PRESSURE - mm MERCURY
TEMPERATURE - DEGREES CENTIGRADE
8500
8000
7000
6000
5000
4000
3000
2000
1000
500
100
110
120
130
140
150
160
170
180
190
200
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
27
28
29
30
31
32

Diagram 1.

tic equation.

The points calculated by means of the new characteristic equation, using the constants above proposed, were found to agree very closely with the experimental results near the middle of the range of temperatures, but at the top of the range, or at 180° and 185° the agreement was not so close, as the new lines here fell below the experimental lines, while on the other hand those at the lower range, or about 110° and 120° fell above.

It was evident, then, that a new set of constants would be necessary to make the agreement uniform throughout the whole range of temperatures. By a study of the constants, the effect on the isothermal lines, which would be produced by making various changes in the constants, was noted. Thus a change in (C) was found to affect the slope of the lines referred to the axis of abscissae. A change in (B) either raised or lowered the isothermals bodily, depending on whether (B) was made larger or smaller than 47.16. The degree of change was greater at the upper than at the lower range of temperature. The constant (a) governs the curvature of the lines, and it was decided that if possible this should be left unchanged, as the lines agreed very well in curvature with the experimental lines. The constants (m) and (n), together with (C), governed the slope, therefore, it was not necessary to change (m) and (n) to get the desired agreement.

It will be seen, then, that the only changes necessary

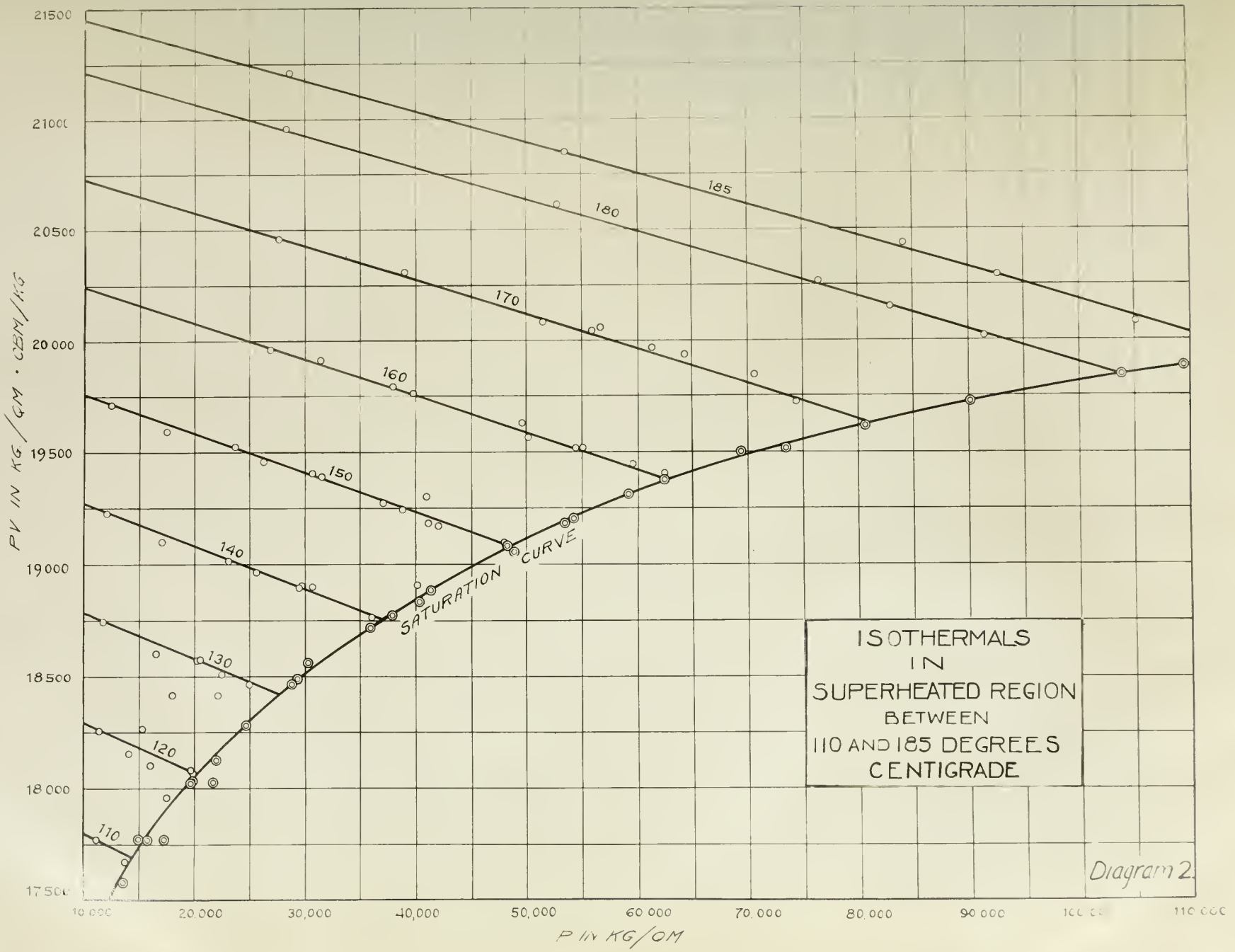

21500
21000
20500
20 000
19500
19000
18500
18 000
17500
PV IN KG/GM · CBM/KG
185
180
170
160
150
140
130
120
110
SATURATION CURVE
ISOTHERMALS
IN
SUPERHEATED REGION
BETWEEN
110 AND 185 DEGREES
CENTIGRADE
10 000
20,000
30,000
40,000
50,000
60 000
70. 000
80,000
90 000
110 000
P IN KG/CM

Diagram 2.

in the constants will be in (B) and (C.)

Various changes and combinations were tried in these two constants, with the end in view of spreading the isothermals apart at the extremes of the range of temperatures until they should agree with the plotted lines. A combination was finally found by means of which a very close agreement was obtained. The values which made this possible were B 47.113 and C .0055. The effect of this change on the equation was to cause the lines at the middle of the range, or 160° which agreed very closely before, to remain stationary, with probably a very slight change in the slope, while the lines at the upper and lower parts of the range spread out and agreed very closely with the experimental lines. The Sample calculations for this operation will be found in Part III..

As in the equation of Knoblauch, Linde and Klebe, the calculated slope $(\frac{dn}{dt})$ in the new equation failed, to some extent, to agree with the slopes observed from Diagram 1. The following table gives a comparison between the observed values and the values calculated from the slope $(\frac{dp}{dt})_V$:

Sp. Vol. Cbm/kg	Saturated Temp.	Sat. Pres. kg/qm observed	Calc.	$(dp/dt)_v$ kg/qm for 1° C obs. K.L.K.	Calc. K.L.K.	Calc. New eq.
1.598	101.4	10860	10860	30.4	30.4	30.9
1.122	112.3	15800	15842	44.3	43.9	44.3
.7405	126.5	24720	24690	68.0	67.4	68.2
.5204	139.2	35960	35980	99.3	97.9	98.2
.3897	150.3	48900	48930	134	132.9	132.5
.2809	163.2	69420	69280	188.6	188.9	187.2
.2428	170	80790	80780	225	221.7	216.7
.1910	180.6	103890	103890	293	287.5	277.7
.1817	183.0	109550	109570	313	303.7	292.0

These slopes were gotten by taking the derivative of p with respect to T of the equation:

$$pv = BT - p(1 + ap)\frac{m}{T}n - Cp.$$

$$\text{Thus } \left(\frac{dp}{dt}\right)_v = \frac{B + \frac{mn}{T^{n+1}} \times p(1 + ap).}{\frac{BT}{P} + ap\frac{m}{T}n}$$

It will be seen that the values of the slopes agree more closely at the higher specific volumes than at the lower, when they are compared with the observed slopes. The discrepancy, however, at the lower range is within the allowable limit of error, as the experimental values, as before stated, are themselves subject to a small error. It would seem possible that as the lines at the lower volume range are very short, there is a chance for error in taking the observed reading of the slope on these short lines. Moreover, a very small error in reading the coordinates of the points will produce a rather large error in the calculated result, and as the difference between the observed and calculated slope grows less as the lines grow longer, both in the case of the values of Knoblauch, Linde and Klebe

and in those of this investigation, it would seem that this inference is probably true.

The new equation may, therefore, be assumed to be sufficiently correct for all practical purposes. It may be seen how close the agreement is by referring to Diagram 2. The small circles on this diagram represent the direct product (pv), while the heavy lines represent the locus of the points calculated from the equation by the method described in Part III. This equation is, moreover, simpler in form that that of Knoblauch, Linde and Klebe and is, therefore, more easily manipulated.

Part III.

Sample Calculations.

Complete Calculations for Conversion of Constants from French to English Units.

It was found desirable to have the constants in English units, as this system is the one most extensively used in this country. The following notation will be used.

T_f = Absolute temperature in French System

T_e = " " " English "

V_f = Volume in French "

V_e = " " English "

P_f = Pressure " French "

P_e = " " English "

With the constants inserted, the equation stands as follows:

$$P_f V_f = 47.16\ T_f - P_f(1+.00000085\ pf)\frac{\text{Antilog}1.19839}{T_f^5} - .006\ P_f$$

Now $T_f = \frac{5}{9} T_e$ where $\frac{5}{9}$ is the temperature conversion factor.

The volume unit is expressed in $^{Cbm}/_{kg}$.

Now 1 meter = 39.37 inches

1 kg = 2.2046 pounds

Hence $cbm/_{kg} = \frac{39.37^3}{1728 \times 2.2046} = 16.018$ $cu.\ ft/_{pound}$.

Therefore the conversion factor for volume is $\frac{1}{16.018}$

The pressure unit is in kg/sq m.

Now $\frac{kg}{sq\ m} = \frac{39.37^2}{144 \times 2.2046} = 4.8825$ $pounds/_{sq.\ ft.}$

The conversion factor for pressure is 4.8825.

The conversion factors for converting units from the French to the English System will be as follows:

$$T_f = \frac{5}{9} T_e$$

$$V_f = \frac{1}{16.018} V_e$$

$$P_f = 4.8825 P_e$$

The equation transformed into English units becomes:

$$\frac{4.8825}{16.018} p_e v_e = 47.16 \times \frac{5}{9} T_e - 4.8825 p_e (1 + .00000085 \times 4.8825 p_e) \frac{\text{Antilog. } 11.19839}{(5/9\ T_e)^5} - (.006 \times 4.8825\ p_e).$$

$$p_e v_e = \frac{47.16 \times 5 \times 16.018\ T_e}{4.8825 \times 9} - 16.018 p_e (1 + .00000085 \times 4.8825 p_e) \frac{\text{Antilog } 11.19839}{(5/9\ T_e)^5} - .006 \times 16.018\ p_e$$

This equation may now be solved for the constants in the English units. In the case of (B) and (C) the constant first proposed and the final value of the constant will both be converted together, as the difference in calculation involves only the changing of the one value in the part of the equation in which it occurs. Let the expressions in the left-hand member of the equation be denoted by (A), (B), (C) for simplicity in reference. Then the equation becomes:

$$p_e v_e = (A) - (B) - (C)$$

Solution of (A) for $B_f = 47.16$

$$(A) = \frac{47.16 \times 5 \times 16.018\ T_e}{9 \times 4.8825}$$

log 47.16 = 1.673574	log 9 = .954243	3.577165	
log 5 = .698970	log 4.8825 = .688642	1.642885	
log 16.018 = 1.204621		1.642885	1.934280 = log 85.957
3.577165			

Solution of (A) for $B_f = 47.113$.

$$\begin{aligned} \log 47.113 &= 1.673141 \\ \text{"} \quad 5 &= .698970 \\ \text{"} \quad 16.018 &= 1.204621 \\ & \quad 3.576732 \\ & \quad 1.642885 \\ & \quad 1.933847 = \log 85.871 \end{aligned}$$

$$B_e = 85.871$$

$$(B). = 16.018 p_e (1 + .00000085 \times 4.8825 p_e) \text{ Antilog } \frac{11.19839}{(5/9\ T_e)\ 5}$$

$$\begin{aligned} \log 16.018 &= 1.204621 \\ & \quad 11.198390 \\ & \quad 12.403011 \\ & \quad 22.403011 - 10 \\ & \quad 8.723635 - 10 \\ & \quad 13.679376 \end{aligned}$$

$$\begin{aligned} \log 5 &= 10.698970 - 10 \\ \log 9 &= .954243 \\ & \quad 9.744727 - 10 \\ & \quad 5 \\ & \quad 48.723635 - 50 \\ & \quad -2.723635 \end{aligned}$$

$\log m_e = 13.679376$

To find new (a)

$$\begin{aligned} \log .00000085 &= -7.929419 \\ \log 4.8825 &= .688642 \\ & \quad -6.618061 = \log .00000415 \end{aligned}$$

$a_e = .00000415$

Solution of (C) for ($C_f = .006$) $.006 \times 16.018 p_e$

$$\begin{aligned} \log .006 &= -3.778151 \\ \log 16.018 &= 1.204621 \\ & \quad -2.982772 = \log .0961 \end{aligned}$$

$C_e = .0961$

Solution of (C) for ($C_f = .0055$) $.0055 \times 16.018 p_e$

$$\begin{aligned} \log .0055 &= -3.740363 \\ \text{"} \quad 16.018 &= 1.204621 \\ & \quad -2.944984 = \log .0881 \end{aligned}$$

$C_e = .0881$

In English units the equation therefore becomes:

$$pv = 85.871\ T + p(1 + .00000415p)\ \frac{\text{Antilog } 13.679376}{T\ 5} - .0881p.$$

Product of Pressure and Volume.

The sample calculations for the product of pressures and volumes will be shown for one experiment only as the method is the same throughout. The final product results in multiplying (p) in $kg/_{sq.m}$ by V in $^{cbm}/_{kg}$.

Let us take for example Exp. 20 with saturation pressure, (p) = 3040, temperature (t) = 144.06 and $\frac{dp}{dt} = 8.3$. This runs from saturation temperature to 185°. The method is as follows:

t	difference	p
144.06	0	3040
150	5.94	3089.3
160	15.94	3172.3
170	25.94	3255.3
180	35.94	3338.3
185	40.94	3379.8

$$\log \frac{dp}{dt} = \log 8.3 = .919078$$
$$\log 5.94 = .773786$$
$$1.692864 = \log 49.302$$

Add to Saturation p, 49.302

The addition of 49.302 is made for the 5.94° increment. For each 10° increment, 10 X 8.3 is added to the last preceding value of pressure, until the 5° increment is reached, when 5 X 8.3 is added to the last preceding pressure. This process gives the pressures as accurately as the values of the slopes are given. Now, having the pressures, we can arrange the calculation in the following form:

$$V = .4566 \quad \log V = \bar{1}.659536$$

p_m pressure in mm. of Hg. p_{kg} pressure in kg/sq m.

conversion factor logarithm of 13.59 which is factor for con-

verting from mm Hg. to kg/sq m.

t	144.06	150	160	170	180	185
p	3040	3089.3	3172.3	3255.3	3338.3	3379.8
log pm.	3.482974	3.489860	3.501374	3.512591	3.523525	3.528891
conv.	1.133412	1.133412	1.133412	1.133412	1.133412	1.133412
log pkg	4.616286	4.623272	4.634786	4.646003	4.656937	4.662303
log v	$\bar{1}$.659536	$\bar{1}$.659536	$\bar{1}$.659536	$\bar{1}$.659536	$\bar{1}$.659536	$\bar{1}$.659536
log pv	4.275822	4.282808	4.294322	4.305539	4.316473	4.321839
pv	18872	19178	19693	20209	20724	20982
p	41332	42002	43131	44259	45388	45952

Calculations for points on lines according to new characteristic equation:

$$pv=BT - p(1+ap)\frac{m}{T}^{n}-CP$$

when: $B = 47.113$ $C = .0055$

$a = .00000085$ $\log m = 11.19839$

$n = 5$

We will take three points on the line 185°, and two on 120° to show the agreement with the isothermal lines derived from the experimental determinations. On the 185° line, we will calculate points on 10000, 50000 and 100000 pressures, and 10000 and 20000 on the 120° isothermal. The equation was divided into parts and arranged as follows:

$t =$ $T = 273.1 + t$

$\log p =$
$\log(1+ap) =$
$\log p(1+ap) =$
add $\log \frac{m}{T^n} =$
log 2nd term =
2nd " =
(Cp) 3rd " =
log B =
log T =
log BT =

2d. & 3d. terms =
pv. =

p(1+ap)	p	(1+ap)	p(1+ap)	log p (1+ap)
	10000	1.0085	10085	4.003676
	20000	1.0170	20340	4.308351
	50000	1.0425	52125	4.717046
	100000	1.0850	108500	5.035430

$\frac{m}{T}\,n$ $\qquad \log \frac{m}{T}\,n = \log m - 5 \log T$

t = 120 T=393.1 $\qquad \log m = 11.19839$

$\log 393.1 = 2.594503 \qquad 5 \log 393.1 = \underline{12.972515}$

$8.225875 - 10 = \log \frac{m}{T}\,n$

t = 185 T=458.1 $\qquad \log m = 11.19839$

$\log 458.1 = 2.660960 \qquad 5 \log 458.1 = \underline{13.304800}$

$7.893590 - 10 = \log \frac{m}{T}\,n$

BT $\qquad \log B = \log 47.113 = 1.673141$

$\log T = \log 393.1 = \underline{2.594503}$

$4.267644 = \log 18520 \qquad BT = 18520$

$\log B = 1.673141$

$\log T = \underline{2.660960}$

$4.334101 = \log 21583 \qquad BT = 21583 \qquad T = 458.1$

CP - $\qquad C = .0055$

p	CP
10000	55
20000	110
50000	275
100000	550

For isothermal 120°

p	10000	20000
$\log p(1+ap)$	4.003676	4.308351
$\log \frac{m}{T}\,n$	-2.225875	-2.225875
log 2nd term	2.229551	2.534226
2nd term.	169.65	342.16
3rd term	55	110
(2 + 3) term	224.65	452.16
1st term	18520	18520.0
pv	18295.35	18067.84

For isothermal 185°

p	10000	50000	100000
log p(1-ap)	4.003676	4.717046	5.035430
log $\frac{m}{T}$ n	-3.893590	-3.893590	-3.893590
log 2d term	1.897266	2.610636	2.929020
2nd term	78.93	407.98	849.22
3rd term	55	275	550
(2-3)rd "	133.93	682.98	1399.22
1st term	21583	21583	21583
	21449.07	20900.02	20183.78

Calculation for slope $(\frac{dp}{dt})_v$.

Let us take for example Experiment 1. t=101.4 p=10860

$$(\frac{dp}{dt})_v = \frac{B + \frac{nm}{Tn+1}}{\frac{BT}{p} + ap\frac{m}{Tn}} \frac{p(1+ap)}{}$$

The expression with values inserted becomes:

$$(\frac{dp}{dt})_v = \frac{47.113 + \frac{5m}{374.5^6}(799 \times 13.596)(1+ap)}{\frac{47.113 \times 374.5}{13.596 \times 799} + \frac{.00000085 \times 13.596 \times 799m}{374.5^5}}$$

Write in the form

$$(\frac{dp}{dt})_v = \frac{A+B}{C+D}$$

Sample calculation Exp. 1.

A = 47.113

Solution of B. log (1-ap) log(a) = -7.929419

log 799 = 2.902547 log pkg = 4.035959

log conv = 1.133412 -3.965378 =

log pkg = 4.035959

log (1+ap) log = 1.0092337 log .0092337

log (1+ap) = .003977

8.8

.003986

log (1+ap) = .003986 6 log 374.5 = 6X2.573452

log 5 = .698970 = 15.440712

log m = 11.198390

log pkg = 4.035959

15.937305

15.440712

log B = .496593 B=3.1376

Solution of C. $\log 47.113 = 1.673141$

$\log 374.5 = 2.573452$

4.246593

$\log pkg = 4.035959$

$\log C = .210634 \quad C = 1.6242$

Solution of D.

$\log a = -7.929419$

$\log pkg = 4.035959$

$\log m = 11.198390$

9.163768

-12.867260

$19.163768 - 10$

12.867260

$6.296508 - 10$

$\log D = -4.296508 \quad D = .00019793$

$5 \log 374.5 = 5 \times 2.573452 = 12.867260$

Final Solution $\frac{(A+B)}{(C+D)} = \frac{47.113 + 3.1376}{1.6242 + .00019793} = \frac{50.2506}{1.62439}$

$\log 50.2506 = 1.701145$

$\log 1.624398 = .210693$

$1.490452 = \log \frac{dp}{dt}$

$\frac{dp}{dt} = 30.935$ kg./ sqm

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